Summary of Effect Sizes and their Links to Inferential Statistics

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Much of this is based on :

Rosenthal, R. (1994). Parametric measures of effect size. In H. Cooper & L. V. Hedges (Eds.), *The handbook of research synthesis* (pp. 231-244). New York: Russell Sage Foundation.

Rosenthal, R. Rosnow, R. L., & Rubin, D. B. (2000) *Contrasts and effect sizes in behavioral research: A correlational approach*. Cambridge, UK: Cambridge University Press.

Please feel free to contact me if any of these formulae seem incorrect – it is possible that typographical errors may have been made.

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1. EFFECT SIZES: DEFINITIONS

1.1 Degree of association between two variables (Correlational Effect Sizes)

$$r = \Phi = r_{pb} = \frac{\sum Z_x Z_y}{n} = \frac{\sum \left(\frac{X-\overline{X}}{\sqrt{\frac{\sum (X-\overline{X})^2}{n}}} \right) \left(\frac{Y-\overline{Y}}{\sqrt{\frac{\sum (Y-\overline{Y})^2}{n}}} \right)}{n} = \frac{\frac{\sum (X-\overline{X})(Y-\overline{Y})}{n}}{\sigma_x \sigma_y} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{s_{xy}}{s_x s_y}$$

NOTE: Some of these formulas use n in the denominator of the correlation, but they are sometimes written with n-1 rather than n in the denominators. This difference does not matter as long as either n or n-1 is used in all parts of the equation (ie, in the standard deviations, z-scores, covariance, etc)

When r is a point-biserial correlation (r_{pb}), it is based on a dichotomous grouping variable (X) and a continuous variable (Y). In this case, the equation can also be written as:

$$r = \frac{\sqrt{p_1 p_2}\left(\overline{Y}_1 - \overline{Y}_2\right)}{\sigma_Y}$$

where p1 and p2 are the proportions of the total sample in each group, $\left(\overline{Y}_1 - \overline{Y}_2\right)$ is the difference between the groups' means on the continuous variable (Y), and σ_Y is the standard deviation of variable Y. Note that this is Equation 5 in McGrath and Meyer (2006), and note that the direction of the correlation depends on which group is considered group 1 and which is considered group 2.

McGrath, R. E., & Meyer, G. J. (2006). When effect sizes disagree: The case of *r* and *d*. ***Psychological Methods*, *11***, 386-401.

Z transformation of a correlation

$$z_r = r' = \tfrac{1}{2}\log_e\left[\frac{1+r}{1-r}\right] = \tfrac{1}{2}\left[\log_e(1+r) - \log_e(1-r)\right]$$

Where $\log_e$ is the natural log function (LN on some calculators)

To transform back from z_r (r') metric to r metric

$$r = \frac{e^{2z_r} - 1}{e^{2z_r} + 1}$$

Where e is the exponent function (e^X on some calculators)

Effect size for the difference between correlations

$$\text{Cohen's } q = Z_{r1} - Z_{r2}$$

1.2 Degree of difference between two means (Effect sizes a la d)

1.2.1 . For comparing means from two groups:

$$\text{Cohen's d} = \frac{\overline{X}_1 - \overline{X}_2}{\sigma_{pooled}}$$

$$\text{Hedges's g} = \frac{\overline{X}_1 - \overline{X}_2}{s_{pooled}}$$

$$\text{Glass's } \Delta = \frac{\overline{X}_1 - \overline{X}_2}{s_{\text{control group}}}$$

Where

$$\sigma_{pooled} = \sqrt{\frac{(n_1)\sigma_1^2 + (n_2)\sigma_2^2}{n_1 + n_2}} \quad \text{and} \quad s_{pooled} = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$\text{and} \quad \sigma_{pooled} = s_{pooled}\sqrt{\frac{n_1 + n_2 - 2}{n_1 + n_2}} = s_{pooled}\sqrt{\frac{N-2}{N}}$$

$$\text{and} \quad s_{pooled} = \frac{\sigma_{pooled}}{\sqrt{\frac{n_1 + n_2 - 2}{n_1 + n_2}}} = \frac{\sigma_{pooled}}{\sqrt{\frac{N-2}{N}}}$$

You may also see σ_{pooled} referred to as σ_{within}, and s_{pooled} referred to as s_{within} or as $\sqrt{MS_{within}}$

1.2.2 The logic of d and g can be applied when comparing one mean to a population mean (e.g., one sample t-test), such that:

$$d = \frac{\overline{X} - \mu}{\sigma_X}$$

$$g = \frac{\overline{X} - \mu}{s_X}$$

where μ is the null hypothesis population mean

1.2.3 The logic of d and g can be applied when comparing two correlated means (e.g., repeated measures t-test, paired samples t-test)

$$d = \frac{\overline{D}}{\sigma_D}$$

$$g = \frac{\overline{D}}{s_D}$$

where $\overline{D}$ is the mean difference score and σ_D and s_D are standard deviations of the difference scores

1.3 "Variance accounted for" R^2, Eta squared (η^2), and omega squared (ω^2)

$$R^2 = \eta^2 = \frac{SS_{EXPLAINED}}{SS_{TOTAL}} = \frac{SS_{BETWEEN}}{SS_{TOTAL}}$$

For a specific effect (ie, in a study with multiple IVs/predictors) $R^2 = \eta^2 = R^2_{EFFECT} = \eta^2_{EFFECT} = \frac{SS_{EFFECT}}{SS_{TOTAL}}$

Omega squared for an effect = $\omega^2_{EFFECT} = \frac{\sigma^2_{EFFECT}}{\sigma^2_{TOTAL}} = \frac{SS_{EFFECT} - df_{EFFECT} MS_{ERROR}}{SS_{TOTAL} + MS_{ERROR}}$

1.4 Effect sizes for proportions

$$\text{Cohen's } g = p - .50$$
where p estimates a population proportion

$$d' = p_1 - p_2$$
where p_1 and p_2 are estimates of the population proportions

$$\text{Cohen's } h = \arcsin p_1 - \arcsin p_2$$

$$\text{Probit } d' = Z_{p_1} - Z_{p_2}$$
where Z_{p_1} and Z_{p_2} are standard normal deviate transformed estimates of population proportions

$$\text{Logit } d' = \log_e\left[\frac{p_1}{1-p_1}\right] - \log_e\left[\frac{p_2}{1-p_2}\right]$$

2. TRANSFORMING BETWEEN EFFECT SIZES

2.1 Computing r

2.1.1 Computing r from Cohen's d

For one group and for two correlated means (ie repeated measures or paired samples)

$$r = \sqrt{\frac{d^2}{d^2 + 1}} = \frac{d}{\sqrt{d^2 + 1}}$$

For two independent groups

$$r = \sqrt{\frac{d^2}{d^2 + \frac{1}{p_1 p_2}}}$$

Where p_1 is the proportion of participants who are in Group 1 and p_2 is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $r = \sqrt{\frac{d^2}{d^2 + 4}}$

2.1.2. Computing r from Hedge's g

For one group and for two correlated means (ie repeated measures or paired samples)

$$r = \sqrt{\frac{g^2}{g^2 + \frac{df}{N}}} = \sqrt{\frac{g^2}{g^2 + \frac{N-1}{N}}}$$

For two independent groups

$$r = \sqrt{\frac{g^2 n_1 n_2}{g^2 n_1 n_2 + (n_1 + n_2)df}} = \sqrt{\frac{g^2}{g^2 + \frac{df}{N p_1 p_2}}} = \sqrt{\frac{g^2}{g^2 + \frac{N-2}{N p_1 p_2}}}$$

Where p_1 is the proportion of participants who are in Group 1 and p_2 is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $r = \sqrt{\frac{g^2}{g^2 + 4\left(\frac{N-2}{N}\right)}}$

2.2 Computing d

2.2.1 Computing d from r

For one group and for two correlated means (ie repeated measures or paired samples)

$$d = \frac{r}{\sqrt{1-r^2}}$$

For two independent groups

$$d = \frac{r}{\sqrt{p_1 p_2 \left(1-r^2\right)}}$$

Where p_1 is the proportion of participants who are in Group 1 and p_2 is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $d = \frac{2r}{\sqrt{1-r^2}}$

2.2.2. Computing d from Hedge's g

For one group and for two correlated means (ie repeated measures or paired samples)

$$d = g\sqrt{\frac{N}{df}} = g\sqrt{\frac{N}{N-1}}$$

For two independent groups (regardless of the relative sample sizes)

$$d = g\sqrt{\frac{N}{df}} = g\sqrt{\frac{N}{N-2}}$$

2.3 Computing g

2.3.1 Computing g from r

For one group and for two correlated means (ie repeated measures or paired samples)

$$g = \frac{r}{\sqrt{1-r^2}}\sqrt{\frac{df}{N}} = \frac{r}{\sqrt{1-r^2}}\sqrt{\frac{N-1}{N}}$$

For two independent groups

$$g = \frac{r}{\sqrt{p_1 p_2 \left(1-r^2\right)}}\sqrt{\frac{df}{N}} = \frac{r}{\sqrt{p_1 p_2 \left(1-r^2\right)}}\sqrt{\frac{N-2}{N}}$$

Where p_1 is the proportion of participants who are in Group 1 and p_2 is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $g = \frac{2r}{\sqrt{1-r^2}}\sqrt{\frac{df}{N}} = \frac{2r}{\sqrt{1-r^2}}\sqrt{\frac{N-2}{N}}$

2.3.2 Computing g from Cohen's d

For one group and for two correlated means (ie repeated measures or paired samples)

$$g = d\sqrt{\frac{df}{N}} = d\sqrt{\frac{N-1}{N}}$$

For two independent groups (regardless of the relative sample sizes)

$$g = d\sqrt{\frac{df}{N}} = d\sqrt{\frac{N-2}{N}}$$

2.4 Transforming between eta squared (η^2) and omega squared (ω^2)

$$\text{eta}^2 \text{ for an effect} = \eta^2 = \frac{\left(df_{effect} + \frac{nk\omega^2}{1-\omega^2}\right)\left(\frac{1}{df_{error}}\right)}{\left(df_{effect} + \frac{nk\omega^2}{1-\omega^2}\right)\left(\frac{1}{df_{error}}\right) + 1}$$

Where n is the number of individuals per group, and k is the number of groups for the effect

$$\text{omega2 for an effect} = \omega^2 = \frac{\frac{df_{error}\eta^2}{1-\eta^2} - df_{effect}}{\left(\frac{df_{error}\eta^2}{1-\eta^2} - df_{effect}\right) + nk}$$

3. COMPUTING SIGNIFICANCE TESTS FROM EFFECT SIZES

Recall, Inferential test statistic = Effect size X Size of Study

3.1 For a 2 x 2 Contingency table

$$\chi^2_{(1)} = Z^2 = r^2 N$$

3.2 For a t-test

3.2.1 T from r

This is appropriate for any kind of t-test:

$$t = \frac{r}{\sqrt{1-r^2}}\sqrt{df}$$

3.2.2 T from Cohen's d

3.2.2.1 For a One-sample t-test or correlated means t-test

$$t = d\sqrt{df} = d\sqrt{N-1} \qquad \textit{where} \quad d = \frac{\bar{X}-\mu}{\sigma_X} \quad \text{or} \quad d = \frac{\bar{D}}{\sigma_D}$$

3.2.2.2 For an independent groups t-test

$$t = d\left(\frac{\sqrt{n_1 n_2}}{(n_1+n_2)}\right)\sqrt{df} = d\sqrt{p_1 p_2 df} = d\sqrt{p_1 p_2 (N-2)}$$

Where p_1 is the proportion of participants who are in Group 1 and p_2 is the proportion in Group 2

Note, for equal sample sizes ($p_1 = p_2 = .50$), this simplifies to: $t = d\frac{\sqrt{df}}{2} = d\frac{\sqrt{N-2}}{2}$

3.2.3 T from Hedge's g

3.2.3.1 For a One-sample t-test or correlated means t-test

$$t = g\sqrt{N} \qquad \textit{where} \quad g = \frac{\bar{X}-\mu}{s_X} \quad \text{or} \quad g = \frac{\bar{D}}{s_D}$$

3.2.3.2 For an independent groups t-test

$$t = g\left(\sqrt{\frac{n_1 n_2}{n_1+n_2}}\right) = g\sqrt{p_1 p_2 N}$$

Where p_1 is the proportion of participants who are in Group 1 and p_2 is the proportion in Group 2

Note, for equal sample sizes ($n_1 = n_2 = n$ and $p_1 = p_2 = .50$), this simplifies to: $t = g\frac{\sqrt{N}}{2} = g\sqrt{\frac{n}{2}}$

3.3 For an ANOVA (F test)

3.3.1 For an ANOVA with $df_{NUMERATOR}$ =1 (two independent groups)

$$F = \frac{r^2}{1-r^2}\left(df_{error}\right)$$

$$F = d^2\left(df_{error}\,p_1 p_2\right)$$

(for equal n study, $F = d^2\left(\frac{df_{error}}{4}\right)$

$$F = g^2\left(\frac{n_1 n_2}{n_1 + n_2}\right) = g^2\left(nkp_1 p_2\right) = g^2\left(Np_1 p_2\right) \text{ for a oneway ANOVA}$$

(for equal n study, $F = g^2\left(\frac{nk}{4}\right) = g^2\left(\frac{N}{4}\right)$ for a oneway ANOVA

$$F = \frac{eta^2}{1-eta^2}\left(df_{error}\right)$$

$$F = \frac{\omega^2}{1-\omega^2}\left(nk\right)+1 = \frac{\omega^2}{1-\omega^2}\left(N\right)+1 \quad \text{for a oneway ANOVA}$$

3.3.2 For an ANOVA with $df_{NUMERATOR}$ > 1 (more than two independent groups)

$$F = \frac{eta^2}{1-eta^2}\left(\frac{df_{error}}{df_{means}}\right)$$

$$F = \frac{\omega^2}{1-\omega^2}\left(\frac{nk}{k-1}\right)+1$$

4. COMPUTING EFFECT SIZES FROM SIGNIFICANCE TESTS

4.1 Computing r

4.1.1 r from a X^2 test for a 2x2 contingency table

$$r = \Phi = r_{pb} = \sqrt{\frac{\chi^2_{(1)}}{n}} = \frac{Z}{\sqrt{n}}$$

4.1.2 r from any t test (or F-test with numerator df = 1)

$$r = \sqrt{\frac{t^2}{t^2 + df}} = \sqrt{\frac{F}{F + df_{error}}}$$

4.2 Computing d

4.2.1 d from a one-sample t-test or correlated means t-test

$$d = \frac{t}{\sqrt{df}} = \frac{t}{\sqrt{N-1}}$$

4.2.2 d from an independent groups t-test

$$d = t\left(\frac{n_1 + n_2}{\sqrt{df}\sqrt{n_1 n_2}}\right) = \frac{t}{\sqrt{p_1 p_2 df_{error}}} = \frac{t}{\sqrt{p_1 p_2 (N-2)}}$$

Which simplifies to $d = \frac{2t}{\sqrt{df}}$ if the groups have equal n

4.2.3 d from an F test based on numerator df = 1

$$d = \frac{\sqrt{F}}{\sqrt{p_1 p_2 df_{error}}} = \frac{\sqrt{F}}{\sqrt{p_1 p_2 (N-2)}}$$

Which simplifies to $d = \frac{2\sqrt{F}}{\sqrt{df}}$ if the groups have equal n

4.3 Computing g

4.3.1 g from a one-sample t-test or correlated means t-test

$$g = \frac{t}{\sqrt{N}}$$

4.3.2 g from an independent groups t-test

$$g = t\sqrt{\frac{n_1 + n_2}{n_1 n_2}} = \frac{t}{\sqrt{p_1 p_2 N}}$$

Which simplifies to $g = \frac{2t}{\sqrt{N}}$ if the groups have equal n

4.3.3 g from an F test based on numerator df = 1

$$g = \frac{\sqrt{F}}{\sqrt{p_1 p_2 N}}$$

Which simplifies to $g = \frac{2\sqrt{F}}{\sqrt{N}}$ if the groups have equal n

4.4 Computing η^2 and ω^2

$$\text{eta}^2 \text{ for an effect} = \eta^2 = \frac{\frac{F_{effect}(df_{effect})}{df_{error}}}{\frac{F_{effect}(df_{effect})}{df_{error}} + 1} = \frac{F_{effect}(df_{effect})}{F_{effect}(df_{effect}) + df_{error}}$$

$$\text{omega}^2 \text{ for an effect} = \omega^2 = \frac{(F_{effect} - 1)(df_{effect})}{(F_{effect} - 1)(df_{effect}) + nk}$$

For F tests with numerator = 1, these simplify to

$$\text{eta}^2 \text{ for an effect} = \eta^2 = \frac{\frac{F_{effect}}{df_{error}}}{\frac{F_{effect}}{df_{error}} + 1} = \frac{F_{effect}}{F_{effect} + df_{error}}$$

$$\text{omega}^2 \text{ for an effect} = \omega^2 = \frac{(F_{effect} - 1)}{(F_{effect} - 1) + nk}$$